Mortgage insurance premiums and business cycle

Chao-Chun Chen* Shih-Kuei Lin† Wen-Shih Chen‡

Abstract

This research first refines the pricing formula for mortgage insurance (MI) contracts proposed in Bardhan, Karapandža, and Urošević (2006) based on a modified contract whose setting is closer to reality. Since housing prices are usually observed to be cyclic in the mean growth rate and volatility due to economic fluctuations, our research further extends this modified MI pricing formula by taking into consideration the state-dependent property behind housing prices. Empirical analysis shows that the fair premium estimated with the state-dependent property for the U.S. market is higher than that when ignoring this property at the end of 2010. It indicates that incorporating the cyclical property in valuing MI contracts may facilitate to lessen the losses for mortgage insurers in the U.S. market, in which these companies operated at a loss during this period. By comparing the market premiums and fair premiums calculated from the proposed model, we find evidence that the high-risk MI insured is largely subsidized by the low-risk insured in the U.S. market.

Key words: Mortgage insurance; Markov regime-switching; Option pricing

JEL classification: G13, G22

*All correspondence to Chao-Chun Chen. Associate Professor. Department of Finance, Tunghai University, Taichung, Taiwan, R.O.C. Tel.: (886-4) 2359-0121 ext. 35809. Fax: (886-4) 2350-6835. E-mail: jawjiun@thu.edu.tw

†Associate Professor. Department of Money and Banking, National Chengchi University, Taipei, Taiwan, R.O.C.

‡Department of Finance, Tunghai University, Taichung, Taiwan, R.O.C.

1 Introduction

Mortgage insurance (MI) contracts provide an important tool for lenders to hedge their risk exposures by insuring mortgages against default. In the U.S. market, more and more lenders hedge their default risk through MI contracts. Figure 1 displays the market values of single-family mortgages outstanding insured by the Federal Housing Administration (FHA) from 1990 to Quarter 2, 2011. We find that insured mortgages have grown explosively after the subprime crisis. Specifically, the amount of insured mortgages rose substantially from \$327.340 billion at the end of 2006 to \$992.132 billion at Quarter 2, 2011, for a growth rate of 200%. Moreover, 91% of the single-family mortgages insured by FHA are fixed-rate mortgages.

The common concern of lenders, borrowers, and insurers with respect to MI contracts is the fair premium of insurance contracts. The critical problem for valuing a MI contract centers on knowing the mortgage termination, which may be caused by either the prepayment or the default on the underlying loan. Generally speaking, prepayment decisions relate to the term structure of the interest rate, while default decisions mainly depend on the housing price. In the literature, Kau, Keenan, Muller, and Epperson (1992, 1995), Kau, Keenan, and Muller (1993), and Kau and Keenan (1995, 1999) value mortgage loans and MI contracts by using dynamic programming methods or backward pricing models. Nevertheless, all of these methods involve complex numerical procedures. As pointed out in Bardhan, Karapandža, and Urošević (2006) (BKU (2006), hereafter), the complexity inherent in these numerical approaches may not be warranted in the consideration of fitting the model to the data.

Instead of developing numerical approaches, BKU (2006) show that the payoff from MI contracts in case of default is equivalent to the payoff from a bear spread created by put options. Accordingly, they develop an option-pricing framework to price MI contracts in closed form. The pricing formula proposed in BKU (2006) not only allows the realized loss for the insurer in case of default to depend on the collateral price and loan balance, but also permits legal inefficiencies to be taken into account.

As the first research that proposes the closed-form solution for valuing MI contracts, the most important advantage inherent in BKU's (2006) approach is ease of use. However, we observe that the setting of BKU's MI contract is not fully coincident with reality. In practice, the insurer's loss in case of default happening at time t is the deficiency between the time t loan balance and collateral price, if the maximum loss defined in MI contracts is not hit. In contrast to the use of time t loan balance, BKU (2006) set the time t conditional loss as the difference between the time $t-1$ value of loan balance and time t collateral price, with a maximum limit as well. This motives us to alter the setting concerning the conditional loss and refine BKU's (2006) formula based on the modified contract, which is the first contribution in this research.

It is well known that the premium of MI contracts depends on the dynamics of housing prices. One common assumption in the literature is to assume the process of housing prices to be a geometric Brownian motion (GBM), though it is unable to capture some important features of housing prices, including time-varying volatility, jump, and state-dependence. In order to incorporate the risk of catastrophic events, Kau and Keenan (1996), Chen, Chang, Lin, and Shyu (2010), and Chang, Huang, and Shyu (2011) investigate the valuation of MI contracts based on jump diffusion processes. In addition to catastrophic events, state-dependence is another property well known in the housing market. A number of empirical studies have provided evidence concerning the state-dependent property inherent in the growth rate of housing prices, including Davis and Heathcote (2005), Clark and Coggin (2009), and Edelstein and Tsang (2007), to name a few.

The content regarding the cyclical property in the volatility of real estate prices is also investigated in research studies. Eichholtz (1997) shows that the behavior of the housing price index, both in terms of price changes and of volatilities, displays different pictures in different time periods. Guirguis, Giannikos, and Anderson (2005) demonstrate that the volatility is cyclical in the U.S. housing market due to structural changes and economic fluctuations. Crawford and Fratantoni (2003) compare the forecasting performance from regime-switching, ARIMA, and GARCH models and find that regime-switching models perform best in-sample for the housing market. Their results indicate that the regime-switching model is a compelling choice for real estate markets that have historically embedded boom and bust cycles. Although BKU's (2006) approach is very easy to apply, the key assumption that the dynamics of housing prices follow the GBM process is far from capturing the state-dependent property behind housing prices at all. This motivates us to fill the gap by extending the pricing formula for MI contracts based on regime-switching option pricing models.

Our study is not the only one to incorporate regime-switching mechanisms into the valuation of MI contracts. In the literature, Lin and Chuang (2010) develop a semi-closed-form approach to value MI contracts under a business cycle. Specifically, Lin and Chuang's (2010) approach is not exactly in closed-form since the unconditional probability of state k occurring j times during the life of options, which is a necessary term in their pricing formula, is calculated through a backward device, while on the contrary the approach proposed in this research is in closed-form. This is the first difference between our research and Lin and Chuang (2010). The second distinguishing feature of our research is to investigate MI contracts with a setting nearer to reality, as we mentioned before. Accordingly, the proposed formula for MI premiums with the cyclical property is developed based on this modified MI contract, rather than the original contract studied in BKU (2006). The third feature of this research different from Lin and Chuang (2010) is the assumption for the dynamic of state-dependent housing prices. In order to capture the state-dependent characteristic, we assume real estate prices to follow the regime-switching option pricing model of Duan, Popova, and Ritchken (2002) (DPR (2002), hereafter). Their model not only allows asset innovations to have feedback effects on volatilities, but also permits regime shift risk to be priced. Although the closed-form solution for pricing MI contracts proposed in our research is based on a two-state uni-directional regime-switching model, the MI premium under a N-state bi-directional regime-switching framework is able to be calculated under DPR's (2002) framework as well.

The remaining parts of this paper are arranged as follows. Section 2 introduces the theoretical framework and refines the MI pricing formula proposed in BKU (2006). Section 3 develops a closed-form solution for MI premiums by taking the state-dependent property underlying housing prices into account. Section 4 conducts numerical analyses to investigate the sensitivity of proposed premiums to transition probabilities. Section 5 empirically estimates the parameters for the U.S. market and provides the fair MI premiums at the end of 2010. Our findings indicate that incorporating the cyclical property of housing prices into the valuation of MI contracts is able to diminish the losses that many mortgage insurers suffered from during this period. Concluding remarks are given in the last section.

2 Theoretical framework of BKU (2006) and the modified MI pricing formula

This research investigates the valuation of MI contracts by incorporating the cyclical feature inherent in housing prices. Before developing our proposed formula for MI premiums, we introduce the setting of the economic environment in the following.

The asset underlying MI contracts is a mortgage loan, in which the borrower provides a property as collateral for a bank loan and promises to transfer the collateral to the lender if he cannot redeem the loan. Although the mortgage is secured by collateral, the lender still is exposed to potential losses due to the possibility that the housing value will be less than the loan balance in case the borrower cannot afford to pay installments.

Denote V_t as the value of collateral at time t. At origination, i.e., $t = 0$, the lender issues a T-period mortgage loan with a fixed mortgage rate c and a loan amount of B_0, where $B_0 = L_V V_0$ and L_V is the loan-to-value ratio. Since mortgage loans are usually redeemed by installments, we assume that the borrower pays an installment y back at each time t, where $t \in (0, T]$. Once the time t installment is paid, the time t loan balance, B_t, is equal to the total present value of the unpaid payments ranging from time $t+1$ to time T:

$$B_t = \frac{y}{c}\left(1 - \frac{1}{(1+c)^{T-t}}\right). \tag{1}$$

As mentioned above, the lender still takes a risk even when mortgage loans are secured by collateral, especially during a housing market depression. Fortunately, insurance markets allow lenders to transfer this risk through MI contracts. According to BKU (2006), the amount that the insurer of a MI contract compensates the lender in case of default occurring at time t is:

$$Loss_t^{BKU} = \max(0, \min(B_{t-1} - V_t, L_R B_{t-1})), \tag{2}$$

where L_R limits the maximum loss for the insurer and is called the loss ratio. Equation (2) clearly indicates that the insurer compensates the lender for the loss when the borrower cannot afford to pay installments. The amount is the difference proceeded from selling the collateral to pay off the mortgage balance, i.e., $B_{t-1} - V_t$, if any. However, the maximum amount paid by the insurer is just the L_R ratio of the loan balance, i.e., $L_R B_{t-1}$. An important contribution of BKU (2006) is to demonstrate that the conditional loss of the insurer, $Loss_t^{BKU}$, can be regarded as a portfolio of two European put options when the borrower's default occurs at time t. It follows that the valuation of MI contracts can be implemented based on the option pricing theorem.

We note that the setting of BKU's contract displayed in Equation (2) is not fully consistent with reality. Specifically, a default happening at time t indicates that the borrower is unable to pay both the time t installment payment y and the remaining $T - t$ future installments, which is worth B_t. It follows that the loan balance when a default happens at time t should be $B_t + y$ totally. Since the insurer of MI contracts bears the loan balance at time t and has the right to possess the underlying collateral in case of default happening at time t, the conditional loss of the insurer at t should be $B_t + y - V_t$, given that the maximum loss is not touched. Accordingly, we modify the setting of BKU's (2006) contract as:

$$Loss_t = \max(0, \min(B_t + y - V_t, L_R(B_t + y))). \tag{3}$$

We note that this set-up is also in line with that behind Equations (19) and (20) of Kau et al. (1995). In the following, we refine BKU's pricing formula based on the modified MI contract defined in Equation (3).

Similar to the result proposed in BKU (2006), Equation (3) indicates that the potential loss borne by the insurer is equivalent to the cash flow from a portfolio of put options:

$$\begin{aligned} Loss_t &= \max(0, \min(B_t + y - V_t, L_R(B_t + y))) \\ &= \max(K_{1,t} - V_t, 0) - \max(K_{2,t} - V_t, 0), \end{aligned} \tag{4}$$

where

$$K_{1,t} = B_t + y,$$

and

$$K_{2,t} = (1 - L_R)(B_t + y).$$

As shown in Equation (4), one of the two options is a long position in a European put option with a strike price of $K_{1,t}$ and a maturity of t, and the other is a short position in a European put option with a strike price of $K_{2,t}$ and a maturity of t. This portfolio is identical to that of BKU (2006) except for strike prices. Particularly, the strike prices under the modified contract defined in Equation (4) are $K_{1,t} = B_t + y$ and $K_{2,t} = (1 - L_R)(B_t + y)$, whereas those in BKU (2006) are $K_{1,t}^{BKU} = B_{t-1}$ and $K_{2,t}^{BKU} = (1 - L_R)B_{t-1}$.

Recognizing that the current value of $Loss_t$, i.e., $\mathcal{L}_0(t)$, can be priced by the option pricing theorem, we follow BKU (2006) to calculate the fair premium of MI contracts, FP_0, by:

$$FP_0 = \sum_{t=1}^{T} P_d(t)\mathcal{L}_0(t), \tag{5}$$

where $P_d(t)$ is the unconditional probability that the borrower defaults at time t.

The value of $\mathcal{L}_0(t)$ displayed in Equation (5) and the fair MI premium FP_0 still depend on the dynamics of housing prices. Assume that the housing price follows the GBM process, i.e.:

$$\frac{dV_t}{V_t} = (\mathcal{U} - \mathcal{S})dt + \Sigma dz_t, \tag{6}$$

where $\mathcal{S}$ denotes the rental yield, $\mathcal{U} - \mathcal{S}$ is the expected annual rate of collateral appreciation, z_t represents a standard Wiener process, and $\Sigma > 0$ is the annual volatility coefficient. Denote $Put(K_{i,t}, t)$ as the value of a put option with a strike price of $K_{i,t}$ and a maturity date of t under the GBM framework, and let $\mathcal{L}_0^{GBM}(t)$ be the current value of $Loss_t$ under the GBM model. According to Equations (4) and (6), the pricing formula for MI contracts proposed in BKU (2006) can be refined as:

$$\begin{aligned}\mathcal{L}_0^{GBM}(t) &\equiv e^{-\mathcal{R}t} E^Q\{\max(K_{1,t} - V_t, 0) \mid \mathcal{F}_0\} - e^{-\mathcal{R}t} E^Q\{\max(K_{2,t} - V_t, 0) \mid \mathcal{F}_0\} \\ &= Put(K_{1,t}, t) - Put(K_{2,t}, t), \end{aligned} \tag{7}$$

where $\mathcal{R}$ is the annual risk-free rate, $E^Q\{\cdot \mid \mathcal{F}_0\}$ denotes the expectation conditional on $\mathcal{F}_0$ under measure Q:

$$Put(K_{i,t}, t) = K_{i,t} e^{-\mathcal{R}t} N(-d_2(K_{i,t})) - V_0 e^{-\mathcal{S}t} N(-d_1(K_{i,t})), \tag{8}$$

$$\begin{aligned} d_1(K_{i,t}) &= \frac{\ln(V_0/K_{i,t}) + (\mathcal{R} - \mathcal{S} + 0.5\Sigma^2)\, t}{\Sigma\sqrt{t}}, \\ d_2(K_{i,t}) &= d_1(K_{i,t}) - \Sigma\sqrt{t}, \qquad \forall\ i = 1, 2, \end{aligned}$$

and $N(\cdot)$ denotes the standard normal cumulative distribution function. The fair premium of the modified MI contract under the GBM framework, FP_0^{GBM}, is then given by the following expression:

$$FP_0^{GBM} = \sum_{t=1}^{T} P_d(t) \mathcal{L}_0^{GBM}(t). \tag{9}$$

As we have mentioned above, the modified formula for MI premiums displayed in Equations (7)-(9) is similar to the original formula of BKU (2006) except for the strike prices of put options. We name the MI premium calculated by the modified formula as the GBM premium in the following.

3 Incorporating the cyclical property in valuing MI contracts

In what follows, we develop the formula for pricing MI contracts by incorporating the state-dependent property of housing prices based on the regime-switching option pricing model proposed in DPR (2002).

Let σ_{t+1}^2 be the conditional variance of the logarithmic return at time t that holds for the period $[t, t+1]$, and r and s denote the risk-free rate and rental yield for each time period, respectively. According to DPR (2002), a two-state regime-switching model can be written as:

$$\ln \frac{V_{t+1}}{V_t} = r + \lambda\sigma_{t+1} - s - \frac{1}{2}\sigma_{t+1}^2 + \sigma_{t+1}\varepsilon_{t+1}, \tag{10}$$

$$\sigma_{t+1} = \begin{cases} \delta_1, & \text{if } F(\varepsilon_t, \xi_t) < \Phi(\sigma_t), \\ \delta_2, & \text{if } F(\varepsilon_t, \xi_t) \geq \Phi(\sigma_t) \end{cases} \tag{11}$$

and

$$\begin{bmatrix} \varepsilon_{t+1} \\ \xi_{t+1} \end{bmatrix} |\mathcal{F}_t \overset{P}{\sim} N(0_{2\times 1}, I_{2\times 2}),$$

where λ stands for the market price of risk, and δ_k $(k = 1, 2)$ denotes the volatility level under state k. The conditional volatility σ_{t+1} is determined by the magnitude of two function values, $F(\varepsilon_t, \xi_t)$ and $\Phi(\sigma_t)$. Herein, both ε_t and ξ_t are standard normal random variables and the random variable ξ_t is independent of ε_t, while $F(\varepsilon_t, \xi_t)$ is a function of ε_t and ξ_t, whereas $\Phi(\sigma_t) \in (0, \infty)$ is a function of time t volatility.

Since the value of $F(\varepsilon_t, \xi_t)$ is allowed to depend on asset innovations ε_t, this model permits asset innovations to have feedback effects on volatility. It also indicates that the asymmetric volatility under this setting is allowed to react with good and bad news in asset returns. DPR (2002) further demonstrate that with the setting of $F(\varepsilon_t, \xi_t) = |\xi_t|$, the feedback mechanism can be switched off and the process defined in Equations (10) and (11) reduces to a uni-directional regime-switching process.

According to Equation (11), the corresponding transition probability matrix with the two-state uni-directional regime-switching model is:

$$\mathcal{P} = \begin{bmatrix} N(\Phi(\delta_1)) - N(-\Phi(\delta_1)) & 1 - N(\Phi(\delta_1)) + N(-\Phi(\delta_1)) \\ N(\Phi(\delta_2)) - N(-\Phi(\delta_2)) & 1 - N(\Phi(\delta_2)) + N(-\Phi(\delta_2)) \end{bmatrix},$$

where $N(\cdot)$ denotes the standard normal cumulative distribution function. To simplify notation and be consistent with the literature concerning Markov-switching models, in which Hamilton (1989) is a representative, we further denote the transition probability matrix as:

$$\mathcal{P} = \begin{bmatrix} N(\Phi(\delta_1)) - N(-\Phi(\delta_1)) & 1 - N(\Phi(\delta_1)) + N(-\Phi(\delta_1)) \\ N(\Phi(\delta_2)) - N(-\Phi(\delta_2)) & 1 - N(\Phi(\delta_2)) + N(-\Phi(\delta_2)) \end{bmatrix} \equiv \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix},$$

where p_{ij} represents the probability of switching to regime j from regime i and $\sum_{j=1}^{2} p_{ij} = 1$ for all i. Please note that the values of $\Phi(\delta_1)$ and p_{11} are corresponding one by one and so are the values of $\Phi(\delta_2)$ and p_{22}. In what follows, we name this two-state uni-directional regime-switching model as the RS model or RS process.

We note that the RS process displayed in Equation (10) is comparable with the GBM process in Equation (6), although the former is in a discrete-time version and the latter is in a continuous-time version. Given the number of regimes to be one, the RS process in Equation (10) reduces to:

$$\ln \frac{V_{t+1}}{V_t} = r + \lambda\sigma - s - \frac{1}{2}\sigma^2 + \sigma\varepsilon_{t+1}. \qquad (10')$$

On the other hand, the discrete-time version for the GBM process in Equation (6) can be written as:

$$\ln \frac{V_{t+1}}{V_t} = \mu - s - \frac{1}{2}\sigma^2 + \sigma\epsilon_{t+1}, \qquad (12)$$

where $\mu \equiv \mathcal{U}\Delta t$, $s \equiv \mathcal{S}\Delta t$, $\sigma \equiv \Sigma\sqrt{\Delta t}$, ϵ_{t+1} is a standard normal random variable, and Δt denotes the time interval between $[t, t+1]$. Recognizing that the drift term μ displayed in Equation (12) equals the sum of the risk-free rate r and risk premium $\lambda\sigma$, i.e., $\mu = r + \lambda\sigma$, the RS process displayed in Equation (10$'$) is equivalent to the discrete-time version of the GBM process displayed in Equation (12) given that the number of regimes is set to be one.

DPR (2002) provide a closed-form solution for European options when the dynamics of the underlying asset follow the RS process, i.e., the two-state uni-directional regime-switching model. Under the RS process, the fair price of a European put option with a strike price $K_{i,t}$, initial volatility δ_k, where $k = 1, 2$, and expiration after t periods can be calculated by:

$$Put(K_{i,t}, t|\sigma_0 = \delta_k) = \sum_{j=0}^{t} \gamma_{t,j|\sigma_0=\delta_k} Put(K_{i,t}, t|\sigma_0 = \delta_k, N = j), \tag{13}$$

where

$$Put(K_{i,t}, t|\sigma_0 = \delta_k, N = j) = K_{i,t}e^{-rt}N(-d_{2j}(K_{i,t})) - V_0e^{-st}N(-d_{1j}(K_{i,t})),$$

$$d_{1j}(K_{i,t}) = \frac{\ln(V_0/K_{i,t}) + rt - st + 0.5\theta_j^2}{\theta_j},$$

$$d_{2j}(K_{i,t}) = d_{1j}(K_{i,t}) - \theta_j,$$

and

$$\theta_j^2 = j\delta_1^2 + (t-j)\delta_2^2, \qquad j = 0, 1, \cdots, t.$$

We denote N here as the number of times switching to state 1 during the remaining life of the option, and $Put(K_{i,t}, t|\sigma_0 = \delta_k, N = j)$ represents the current price of the European put option under the RS model, given that initial volatility is δ_k and the number of times switching to state 1 during the remaining life of the option is j times. The probability

$\gamma_{t,j|\sigma_0=\delta_k}$ represents that in t periods the number of times for visiting state 1 is j. It can be calculated as follows:

$$\gamma_{t,j|\sigma_0=\delta_1} = \begin{cases} p_{12}p_{22}^{t-1}, & \text{for } j=0 \text{ and } t=1,2,\cdots,T, \\ p_{11}, & \text{for } j=1 \text{ and } t=1, \\ p_{11}p_{12}p_{22}^{t-2}+(t-2)p_{12}^2p_{21}p_{22}^{t-3}+p_{12}p_{21}p_{22}^{t-2}, & \text{for } j=1 \text{ and } t=2,3,\cdots,T, \\ \sum_{i=1}^{t-j+1} F(i|\sigma_0=\delta_1)\gamma_{t-i,j-1|\sigma_i=\delta_1}, & \text{for } j=2,3,\cdots,t, \text{ and } t=2,3,\cdots,T, \end{cases}$$

and

$$\gamma_{t,j|\sigma_0=\delta_2} = \begin{cases} p_{22}^t, & \text{for } j=0 \text{ and } t=1,2,\cdots,T, \\ p_{21}, & \text{for } j=1 \text{ and } t=1, \\ (t-1)p_{21}p_{12}p_{22}^{t-2}+p_{22}^{t-1}p_{21}, & \text{for } j=1 \text{ and } t=2,3,\cdots,T, \\ \sum_{i=1}^{t-j+1} F(i|\sigma_0=\delta_2)\gamma_{t-i,j-1|\sigma_i=\delta_1}, & \text{for } j=2,3,\cdots,t, \text{ and } t=2,3,\cdots,T, \end{cases}$$

where

$$F(i|\sigma_0=\delta_1) = \begin{cases} p_{11}, & \text{for } i=1, \\ p_{12}p_{22}^{i-2}p_{21}, & \text{for } i=2,3,\cdots,t, \end{cases}$$

and

$$F(i|\sigma_0=\delta_2) = \begin{cases} p_{21}, & \text{for } i=1, \\ p_{22}^{i-1}p_{21}, & \text{for } i=2,3,\cdots,t. \end{cases}$$

We define $F(i|\sigma_0 = \delta_1)$ and $F(i|\sigma_0 = \delta_2)$ above as the probability that the first transition to state 1 occurs after i periods given that the initial regime is state 1 and 2, respectively. The probability $\gamma_{t-i,j-1|\sigma_i=\delta_1}$ indicates that in the remaining $t-i$ periods of the option's life the number of times state 1 turns up is $j-1$ times given that the time i regime is state 1. The value of $\gamma_{t-i,j-1|\sigma_i=\delta_1}$ is equal to $\gamma_{t-i,j-1|\sigma_0=\delta_1}$, because the randomness of state-switching from time i to t equals that from time 0 to $t-i$ as long as the initial states are identical, i.e., $\sigma_i = \delta_1$ and $\sigma_0 = \delta_1$, respectively.

Based on the RS option pricing formula, we are able to incorporate the cyclical property inherent in housing prices into the valuation of MI contracts. According to Equations

(4) and (13), the current value of the time t conditional loss to insurers, given that the current state is in regime k, can be valued by:

$$\begin{aligned}\mathcal{L}_0^{RS}(t|\sigma_0=\delta_k) &\equiv Put(K_{1,t},t|\sigma_0=\delta_k) - Put(K_{2,t},t|\sigma_0=\delta_k)\\ &= \sum_{j=0}^{t}\gamma_{t,j|\sigma_0=\delta_k}[K_{1,t}e^{-rt}N(-d_{2j}(K_{1,t})) - V_0e^{-st}N(-d_{1j}(K_{1,t}))]\\ &\quad - \sum_{j=0}^{t}\gamma_{t,j|\sigma_0=\delta_k}[K_{2,t}e^{-rt}N(-d_{2j}(K_{2,t})) - V_0e^{-st}N(-d_{1j}(K_{2,t}))], \end{aligned} \quad (14)$$

where

$$d_{1j}(K_{i,t}) = \frac{\ln(V_0/K_{i,t}) + rt - st + 0.5\theta_j^2}{\theta_j},$$

$$d_{2j}(K_{i,t}) = d_{1j}(K_{i,t}) - \theta_j, \quad i = 1,2,$$

$$K_{1,t} = B_t + y,$$

and

$$K_{2,t} = (1 - L_R)(B_t + y).$$

Denote the probabilities that the initial regime is in states 1 and 2 as $P(\sigma_0 = \delta_1)$ and $P(\sigma_0 = \delta_2)$, respectively. Since the initial state may be in either state 1 or 2, the MI premium under the RS framework, FP_0^{RS}, is given by the following expression:

$$FP_0^{RS} = \sum_{t=1}^{T} P_d(t)\left\{P(\sigma_0=\delta_1)\mathcal{L}_0^{RS}(t|\sigma_0=\delta_1) + P(\sigma_0=\delta_2)\mathcal{L}_0^{RS}(t|\sigma_0=\delta_2)\right\}. \quad (15)$$

4 Numerical analysis

This section conducts numerical analysis to investigate the sensitivity of RS premiums to transition probabilities. To be consistent with market convention, we report the fair premium on an annual-paid basis and represent this annual-paid premium as a ratio of the underlying mortgage loan B_0 in the following. This ratio is called Equivalent Annual Premium (EAP). Since an annual-paid premium terminates once the underlying mortgage loan defaults, a reasonable return required by the insurer for transferring the fair MI premium from a lump-sum-payment basis to an annual basis is the mortgage contract rate, c. Accordingly, the way to transfer the fair MI premium to EAP is given by:

$$EAP = \frac{FP_0^i}{B_0} \times \frac{c}{(1+c)(1-\frac{1}{(1+c)^T})}. \tag{16}$$

Here, FP_0^i represents the fair premium calculated from model i, which is the GBM or RS model.

Table 1 exhibits the sensitivity of EAP calculated from the RS model to transition probabilities, i.e., p_{11} and p_{22}. Parameters in Table 1 are given by: $T = 30$, $L_V = 0.9$, $c = 5\%$, $\mathcal{R} = 0.5\%$, $L_R = 0.75$, $P_d(t) = 0.02$, $\mathcal{S} = 5\%$, $V_0 = 1,000,000$, $\delta_1 = 1\%$, and $\delta_2 = 3\%$. We note that under this setting, state 1 indicates the low-volatility state, whereas state 2 represents the high-volatility state.

Given the initial regime is in state 1, the case of $p_{11} = 1$ indicates that volatility always remains in the low-variance state and thus the MI premium reduces to the price under the GBM model. The characteristic is clearly born out in Table 1. We find that all entries in the last row of Panel A are 77.00 bps, where 1bp is equal to 0.01%. This value is not only the lowest value observed from Panel A of Table 1, but also equals the GBM premium calculated with the setting of $\sigma = 1\%$, as displayed in the footnote of Table 1. Moreover, given the initial regime as state 1 and $p_{22} = 1$, the variance regime stays in regime 2 forever once it leaves regime 1. The time that the variance stays in the low-volatility regime shortens as the value of p_{11} declines. Therefore, for the case of $p_{22} = 1$ and when the initial regime is in regime 1, the smaller the value of p_{11} is, the larger the RS premium will be. This is what we observed from the last column of Panel A in Table 1. Similar characteristics are shared within the cases of Panel B as well, in which the initial regime is in state 2.

For any given value of p_{22} and the initial regime, Table 1 also demonstrates that the RS fair premium is non-increasing as the value of p_{11} grows, because the probability of switching to the low-volatility state increases with p_{11}. Similarly, given the value of p_{11} and the initial regime, the probability of switching to the high-volatility regime increases as the value of p_{22} grows. This is the reason why the MI premium displays a non-decreasing function of p_{22} when the value of p_{11} and the initial regime are given.

Table 1 also shows that the fair MI premium varies with different initial states even when other parameters remain unchanged. To illustrate, the corresponding cases of $p_{11} = 0.25$ and $p_{22} = 0.25$ in Panels A and B produce different MI premiums even though all parameters are identical except the initial regime. To investigate how the MI premium varies with different initial states, the randomness of state-switching plays an important role. Specifically, the randomness of state-switching for any two corresponding cases in Panels A and B is equivalent from time 2 until maturity, but differs solely in the first time interval. For the first time interval, the probability of switching to the high-volatility state is p_{22} given the initial state is regime 2, whereas this probability is p_{12} if the initial state is in regime 1. The probability of switching to the high-volatility state thus results in the fair RS premium initially in the high-volatility state to be higher than that under the low-volatility regime as long as $p_{22} > p_{12}$. On the contrary, the insurer that faces a housing market with transition probabilities of $p_{12} > p_{22}$ should charge a higher premium in the low-volatility state than that in the high-volatility regime based on the RS model. Table 1 shows this phenomenon, which indicates that in the event of $p_{22} > p_{12}$, as with the case of the U.S. housing market at the end of 2010 where we will investigate in the following, an insurer that charges a constant MI premium may suffer a loss when housing prices are at the high-volatility state, but gain a profit when they are in the low-volatility state.

5 Empirical analysis

In this section we empirically value MI contracts for the U.S. market based on the proposed pricing formulae from the GBM model and RS model and then compare the GBM premiums and RS premiums with the Annual Mortgage Insurance Premiums of the Federal Housing Administration (FHA). The data, all-transactions house price index in the U.S. market, come from the Federal Housing Finance Agency (FHFA) and range from Quarter 1, 1975 to Quarter 4, 2010 (144 observations). Moreover, the returns of the house price indices, dV_t/V_t, are calculated as the percentage change in the house price indices.

Table 2 reports the descriptive statistics of returns in the U.S. housing market. The returns of the housing price indices have a mean of 1.2232% and a standard deviation of 1.2107%. The skewness coefficient of -0.2015 implies that the distribution of U.S. housing returns skews left, whereas the kurtosis coefficient of more than 3 indicates that it is a peaked distribution. Moreover, the minimum and maximum are -2.5371% and 4.7331%, respectively.

In order to obtain the parameters of MI pricing formulae, we empirically estimate the parameters of the RS model and GBM model for the U.S. housing market by using the maximum likelihood estimation. Table 3 summarizes the results. In order to simplify notations, we denote $\mu_k \equiv r + \lambda\delta_k$, where $k = 1, 2$, for the RS model and rewrite the drift term of the RS model in Equation (10) as $(\mu_k - s)$ in the following. As shown in Table 3, the expected return $(\widehat{\mu} - s)$ and volatility $\widehat{\sigma}$ of the GBM model are estimated respectively as 1.2232% and 1.2064%. On the other hand, the expected returns of the house price indices estimated from the two-state RS model, $(\widehat{\mu}_1 - s)$ and $(\widehat{\mu}_2 - s)$, are 1.1213% and 1.3985%, whereas the estimated volatility for each state, $\widehat{\delta}_1$ and $\widehat{\delta}_2$, are 0.6416% and 1.7894%, respectively. According to the information list on the website of Global Property Guide, the annual housing yield $\mathcal{S}$ is 5%, which corresponds to a quarterly yield of $s = 1.227\%$. Accordingly, the parameters $\widehat{\mu}$, $\widehat{\mu}_1$, and $\widehat{\mu}_2$ can be calculated easily. Since the volatility of state 2 is larger than that of state 1, regime 2 is regarded as the high-volatility state and regime 1 is the low-volatility state. Nevertheless, the two parameters of the GBM model, $\widehat{\mu}$ and $\widehat{\sigma}$, are both observed to lie in-between their corresponding state-dependent parameters - that is, $(\widehat{\mu}_1, \widehat{\mu}_2)$ and $(\widehat{\delta}_1, \widehat{\delta}_2)$.

Figure 2 displays the time series plot for returns in the housing price indices and the inferred probability of being in state 2. We find that the inferred probability of state 2 is close to 1 when the housing market is volatile, but approaches to 0 when the market is relatively stable. This phenomenon is in line with parameters estimated from the RS model, in which the estimated volatility of state 2 is greater than that of state 1. We also note that the RS model clearly dates the regime transferring to the high-volatility state in the beginning of the subprime crisis, i.e., the third Quarter of 2007. It indicates that the ability of the RS model in capturing the cyclical-volatility property of housing prices is superior.

Once the input parameters are estimated from the U.S. housing price indices covering the period from Quarter 1, 1975 to Quarter 4, 2010, the EAP under the RS model and GBM model at the end of 2010 are able to be calculated based on the proposed pricing formulae. Table 4 displays the EAP at the end of Quarter 4, 2010. The parameters concerning the calculation of RS premiums are consistent with the estimations list in Table 3 and summarized as follows:

$$\widehat{\delta}_1 = 0.6416\%, \quad \widehat{\delta}_2 = 1.7894\%, \quad \widehat{p}_{11} = 0.9823, \quad \widehat{p}_{22} = 0.9856,$$

$$P(\sigma_0 = \delta_1) = 0.0046, \quad P(\sigma_0 = \delta_2) = 0.9954,$$

whereas the volatility for calculating GBM premiums, $\widehat{\sigma}$, as shown in Table 3 is 1.2064%.

Other common parameters are given by:

$$V_0 = 1,000,000,\ (T, c) = \{(15, 4.2\%), (30, 4.71\%)\},\ L_R = 0.75,\ \mathcal{R} = 0.438\%,\ \mathcal{S} = 5\%,$$

in which we adopt the 1-year zero yield observed on December 31, 2010, i.e., the end of our data period, as the risk-free interest rate $\mathcal{R}$ and employ the 15-year and 30-year fixed mortgage contract rates in December 2010 as the 15-year and 30-year mortgage rates, c, respectively. Both the 1-year zero yield and mortgage rates are obtained from the Datastream Database. The figure of the housing yield $\mathcal{S}$ comes from the information list on the website of Global Property Guide. Without loss of generalization, we follow the design in Section 4.1 of BKU (2006) to set the loss ratio L_R as 75%. We note that MI premiums are found to be rarely sensitive when the value of L_R lies in [0.36, 1], given that the other parameters remain in the current setting.

Table 4 displays the fair EAP for the U.S. market under various loan-to-value ratios L_V and maturities T. For the purpose of comparison, we investigate three scenarios concerning the unconditional default probability $P_d(t)$: the low-probability scenario, average-probability scenario, and high-probability scenario. In the average-probability scenario, the setting of $P_d(t) = 1.8\%$ is born from averaging the fixed-rate mortgage delinquency rates of FHA, which are listed in the Datastream Database and cover the period from Quarter 1, 1997 to Quarter 4, 2010. The $P_d(t)$ adopted in the low-probability scenario is the average of mortgage delinquency rates before the subprime crisis, i.e., the period from Quarter 1, 1997 to Quarter 2, 2007, whereas the probability for the high-probability scenario is the average delinquency rate during the subprime crisis, which is the time from Quarter 3, 2007 to Quarter 4, 2010.

As expected, both the RS premium and GBM premium in Table 4 grow with the increase in the loan-to-value ratio L_V, maturity T, and unconditional default probability $P_d(t)$. This is because the risk exposures of insurers increase when one of the three factors, L_V, T, and $P_d(t)$, grows. We also observe that the RS model suggests a higher EAP than the GBM model does at the end of 2010. Based on the findings of the RS model displayed in Table 1, the insurer that faces a housing market with transition probabilities of $p_{22} > p_{12}$ should charge more premiums given that it is currently in the high-volatility state. Since the estimated transition probability $\widehat{p}_{22} = 0.9856$ from the U.S. market is far larger than the estimate of $\widehat{p}_{12}$, the RS model thus suggests insurers in the U.S. market to charge a higher premium when the housing market is volatile, which is also the scenario at the end of 2010, but charge a lower premium when the market is in a low-volatility state.

During 2008-2010, many mortgage insurers were reported to have suffered great losses on most types of loans in the websites of CNBC, HousingWire, and Bloomberg. To illustrate, mortgage insurer Radian Group (RDN) announced that losses for 2010 hit \$1.8 billion, ballooning from \$147.9 million in losses for 2009. In November 2010, mortgage insurer PMI Group (PMI) reported a loss of \$281.1 million in the third quarter of 2010 and the company set aside more funds for potential losses. Old Republic International Corporation (ORI), which provides residential mortgage insurance as well as title insurance and other real estate transfer-related services, reported a quarterly net loss of \$126.5 million in 2009 and warned of a continued slump into 2010. As mentioned above, many insurance companies operated at losses after the subprime crisis. Based on the empirical results that the RS premium is higher than the GBM premium and the RS model suggests a larger premium when the U.S. housing market is volatile, incorporating the cyclical property of housing prices in valuing MI contracts may facilitate to reduce the losses of American insurers, as commonly observed during the crisis.

As Mortgagee Letters proclaimed on February 14, 2011, FHA announced a 25-bp increase in the Annual Mortgage Insurance Premium. The increase in premium is effective for case numbers assigned on or after April 18, 2011. Table 4 displays the Annual Mortgage Insurance Premiums reported by FHA in both the two terms, i.e., through April 17, 2011 and on or after April 18, 2011. By comparing the RS premium with FHA's premium, we find the phenomenon that the high-risk insured, i.e., the high-L_V insured and long-term insured, is large heavily subsidized by the low-risk insured.

We also find that most long-term insurances suffer losses before this 25-bp increase in the Annual Mortgage Insurance Premium. To illustrate, based on the FHA's premium through April 17, 2011, Table 4 indicates that many insurances, especially for the long-term and high-L_V insurances, result in losses to insurers at the end of 2010 even though the unconditional default probability $P_d(t)$ stays at the average of the whole period ranging from Quarter 1, 1997 to Quarter 4, 2010. That may be the reason why FHA required a 25-bp increase in the Annual Mortgage Insurance Premium after April 18, 2011. Nevertheless, given that the unconditional default probability $P_d(t)$ remains at the high probability level of $P_d(t) = 0.023$, i.e., the average level of the subprime crisis, the findings from the RS model indicate that some high-L_V and long-term insurances still lead to losses after a 25-bp increase in the Annual Mortgage Insurance Premium.

6 Conclusions

This paper first modifies the setting of conditional losses to insurers for MI contracts studied by BKU (2006) in order to make the set-up closer to reality and refines the MI pricing formula proposed in BKU (2006) based on the modified contract. To incorporate the important characteristic in housing prices whereby both the mean growth rate and volatility are cyclical, we further extend the pricing formula for MI contracts based on the RS model proposed in DPR (2002).

According to results of numerical analysis, the RS premium not only depends on volatility, but also varies with transition probabilities, p_{11} and p_{22}, and the current regime. For the housing market with transition probabilities of $p_{22} > p_{12}$, where $\delta_2 > \delta_1$, a distinguishable characteristic of the RS model is to suggest that insurers charge a higher premium when the housing market is in the high-volatility state, such as the case of the U.S. housing market at the end of 2010, but charge a lower premium under the low-volatility state. We also empirically value MI contracts for the U.S. market at the end of 2010. Based on empirical results that the RS premium is higher than the GBM premium and the RS model suggests a larger premium when the U.S. housing market is volatile, incorporating the cyclical property into the valuation of MI contracts is helpful for reducing losses of American mortgage insurers at the end of 2010.

To the best of our knowledge, no paper prior to this research compares the fair MI premiums with the FHA premiums. This research compares the fair RS premiums with the Annual Mortgage Insurance Premiums of FHA and finds that the high-risk MI insured is largely subsidized by the low-risk insured in the U.S. market. It is expected that the proposed pricing formula for MI contracts may facilitate further studies in investigating the structure of premiums.

References

1. Bardhan, A., R. Karapandža, and B. Urošević, 2006, Valuing Mortgage Insurance Contracts in Emerging Market Economies, *Journal of Real Estate Finance and Economics*, 32, 9-20.

2. Chang, C. C., W. Y. Huang, and S. D. Shyu, 2011, Pricing Mortgage Insurance with Asymmetric Jump Risk and Default Risk: Evidence in the U.S. Housing Market, *Journal of Real Estate Finance and Economics*

3. Chen, M. C., C. C. Chang, S. K. Lin, and S. D. Shyu, 2010, Estimation of Housing Price Jump Risks and their Impact on the Valuation of Mortgage Insurance Contracts *The Journal of Risk and Insurance*, 77, 399-422.

4. Clark, S. P., and T. D. Coggin, 2009, Trends, Cycles and Convergence in U.S. Regional House Prices, *Journal of Real Estate Finance and Economics*, 39, 264-283.

5. Crawford, G. W., and M. C. Fratantoni, 2003, Assessing the Forecasting Performance of Regime-Switching, ARIMA and GARCH Models of House Prices, *Real Estate Economics*, 31, 223-243.

6. Davis, M. A., and J. Heathcote, 2005, Housing and the Business Cycle, *International Economic Review*, 46, 751-784.

7. Duan, J. C., I. Popova, and P. Ritchken, 2002, Option Pricing under Regime Switching, *Quantitative Finance*, 2, 1-17.

8. Edelstein, R. H., and D. Tsang, 2007, Dynamic Residential Housing Cycles Analysis, *Journal of Real Estate Finance and Economics*, 35, 295-313.

9. Eichholtz, P. M., 1997, A Long Run House Price Index: The Herengracht Index, 1628-1973, *Real Estate Economics*, 25, 175-192.

10. Guirguis, H. S., C. I. Giannikos, and R. I. Anderson, 2005, The US Housing Market: Asset Pricing Forecasts Using Time Varying Coefficients, *Journal of Real Estate Finance and Economics*, 30, 33-53.

11. Hamilton, J. D., 1989, A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica*, 57, 357-384.

12. Kau, J. B., and D. C. Keenan, 1995, An Overview of the Option-Theoretic Pricing of Mortgages, *Journal of Housing Research*, 6, 217-244.

13. Kau, J. B., and D. C. Keenan, 1996, An Option-Theoretic Model of Catastrophes Applied to Mortgage Insurance, *The Journal of Risk and Insurance*, 63, 639-656.

14. Kau, J. B., and D. C. Keenan, 1999, Catastrophic Default and Credit Risk for Lending Institutions, *Journal of Financial Services Research*, 15, 87-102.

15. Kau, J. B., D. C. Keenan, and W. J. Muller, 1993, An Option-Based Pricing Model of Private Mortgage Insurance, *Journal of Risk and Insurance*, 60, 288-299.

16. Kau, J. B., D. C. Keenan, W. J. Muller, and J. F. Epperson, 1992, A Generalized Valuation Model for Fixed-Rate Residential Mortgages, *Journal of Money, Credit, and Banking*, 24, 279-299.

17. Kau, J. B., D. C. Keenan, W. J. Muller, and J. F. Epperson, 1995, The Valuation at Origination of Fixed-Rate Mortgages with Default and Prepayment, *Journal of Real Estate Finance and Economics*, 11, 5-36.

18. Lin, S. K., and M. C. Chuang, 2010, The Valuation of Mortgage Insurance Contracts under Business Cycles: Evidences from Housing Price Index. Master thesis, Department of Finance, National University of Kaohsiung.

Table 1. Sensitivity of Equivalent Annual Premiums (EAP) calculated from the RS model to transition probabilities

(**basis points**)

Panel A: Given the initial regime is state 1.

		p_{22}				
		0.0	0.25	0.5	0.75	1.0
p_{11}	0.0	90.44	92.23	94.54	97.66	102.07
	0.25	88.54	90.36	92.83	96.35	101.84
	0.5	85.97	87.69	90.20	94.16	101.37
	0.75	82.30	83.61	85.73	89.73	99.98
	1.0	77.00	77.00	77.00	77.00	77.00

Panel B: Given the initial regime is state 2.

		p_{22}				
		0.0	0.25	0.5	0.75	1.0
p_{11}	0.0	90.03	91.89	94.29	97.51	102.07
	0.25	88.18	90.09	92.67	96.35	102.07
	0.5	85.68	87.53	90.20	94.42	102.07
	0.75	82.12	83.61	86.02	90.54	102.07
	1.0	77.00	77.29	77.87	79.45	102.07

Note: (1) The parameters in Table 1 are given by: T = 30, L_V = 0.9, c = 5%, $\mathcal{R}$ = 0.5%, $L_R = 0.75$, $P_d(t) = 0.02$, S = 5%, $V_0 = 1{,}000{,}000$, $\delta_1 = 1\%$, and $\delta_2 = 3\%$.

(2) The fair premium under the RS model is calculated by Equations (13)-(15). In order to make a comparison with the market quotes, we transfer the lump-sum-payment premium to an annual-paid basis, i.e., EAP, by Equation (16).

(3) EAP is quoted in basis points, where 1bp is equal to 0.01%.

(4) Based on Equations (7)-(9), the GBM price with the setting of σ = 1% is 77.00 bps, whereas that with the setting of σ = 3% is 102.07 bps. As shown in Table 1, the EAP from the RS model in the case when the initial state is regime 1 and p_{11} = 1 is identical to the GBM premium under σ = 1%, while the RS price given that the initial state is regime 2 and p_{22} = 1 equals the GBM price under σ = 1%.

Table 2. Descriptive statistics of returns in the U.S. housing market

(%)

	Mean	Standard Deviation	Skewness	Kurtosis	Minimum	Maximum
U.S.	1.2232	1.2107	-0.2015	4.4913	-2.5371	4.7331

Note: The data, all-transactions house price index in the U.S. market, are obtained from the Federal Housing Finance Agency (FHFA). The sample period ranges from Quarter 1, 1975 to Quarter 4, 2010, and thus there are 144 observations.

Table 3. Parameter estimates of the RS model and GBM model

	$\mu_1 - s$	$\mu_2 - s$	p_{11}	p_{22}	δ_1	δ_2	$\mu - s$	σ
RS	1.1213	1.3985	0.9823	0.9856	0.6416	1.7894		
	(0.0715)	(0.2502)	(0.0143)	(0.0172)	(0.0526)	(0.1850)		
GBM							1.2232	1.2064
							(0.1009)	(0.0713)

Note: The data, all-transactions house price index in the U.S. market, are obtained from the Federal Housing Finance Agency (FHFA). The sample period ranges from Quarter 1, 1975 to Quarter 4, 2010. Figures in parentheses are standard deviations.

Table 4. Equivalent Annual Premiums (EAP) estimated from the U.S. market

(**basis points**)

		Scenario 1: Low probability $P_d(t)=0.017$		Scenario 2: Average probability $P_d(t)=0.018$		Scenario 3: High probability $P_d(t)=0.023$		Annual Mortgage Insurance Premiums by FHA	
T	L_V	GBM	RS	GBM	RS	GBM	RS	Through 4/17/2011	On/After 4/18/2011
15	0.875	0.00	0.04	0.00	0.05	0.00	0.06	None	25
	0.900	0.01	0.15	0.01	0.16	0.01	0.20		
	0.925	0.05	0.44	0.06	0.46	0.07	0.59	25	50
	0.950	0.30	1.16	0.32	1.23	0.41	1.57		
	0.975	1.32	2.77	1.40	2.93	1.79	3.75		
30	0.875	50.63	54.40	53.61	57.60	68.50	73.60	85	110
	0.900	62.72	65.96	66.41	69.84	84.86	89.24		
	0.925	75.59	78.26	80.03	82.86	102.26	105.88		
	0.950	89.05	91.16	94.29	96.52	120.48	123.33		
	0.975	102.97	104.53	109.03	110.68	139.32	141.43	90	115

Note: (1) We calculate the EAP at the end of 2010 based on estimates displayed in Table 3 and data from the U.S. market. Specifically, we adopt the 1-year zero rate as the risk-free interest rate $\mathcal{R}$, which is 0.438%, and employ the 15-year and 30-year fixed mortgage contract rates as the 15-year and 30-year mortgage rates c, which are 4.2% and 4.71%, respectively. According to the information on the website of Global Property Guide, the U.S. housing yield $\mathcal{S}$ is set to be 5%. Other parameters are given by: V_0 =1,000,000 and $L_R = 0.75$.

(2) Based on Table 3, parameters concerning the RS premiums are: $\hat{\delta}_1 = 0.6416\%$, $\hat{\delta}_2 = 1.7894\%$, $\hat{p}_{11} = 0.9823$, $\hat{p}_{22} = 0.9856$, $P(\sigma_0 = \delta_1) = 0.0046$, and $P(\sigma_0 = \delta_2) = 0.9954$. The parameter for the GBM premiums is $\hat{\sigma} = 1.2064\%$..

(3) The figure of $P_d(t) = 1.825\%$ in the average-probability scenario is born of averaging the fixed-rate mortgage delinquency rates from FHA, covering the period from Quarter 1, 1991 to Quarter 4, 2010. The value of $P_d(t)$ adopted in the low-probability scenario is the average of delinquency rates before the subprime crisis, i.e., the period from Quarter 1, 1997 to Quarter 2, 2007, whereas $P_d(t)$ for the high-probability scenario is the average rate during the subprime crisis, which is the time from Quarter 3, 2007 to Quarter 4, 2010. The Annual Mortgage Insurance Premiums of FHA come from Mortgage Letter on February 14, 2011.

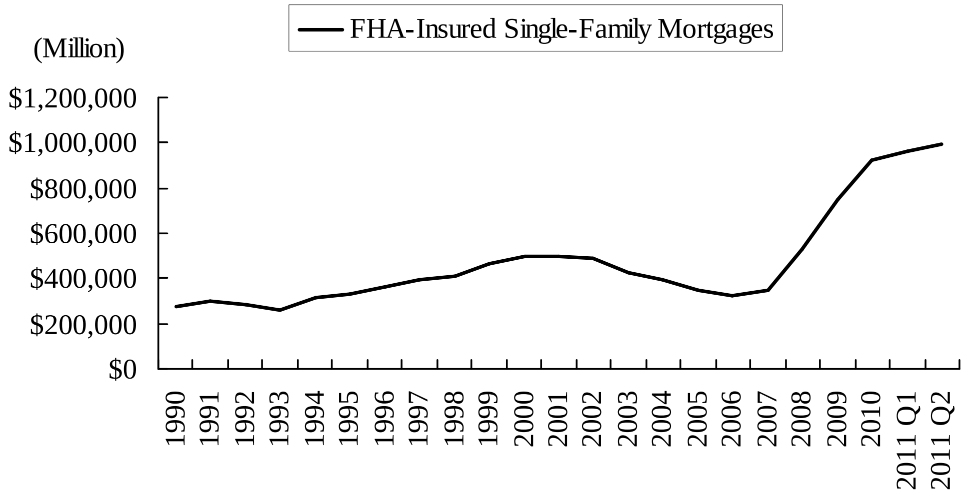

Figure 1. FHA-insured single-family mortgages outstanding, ranging from 1990 to 2011(Q2).

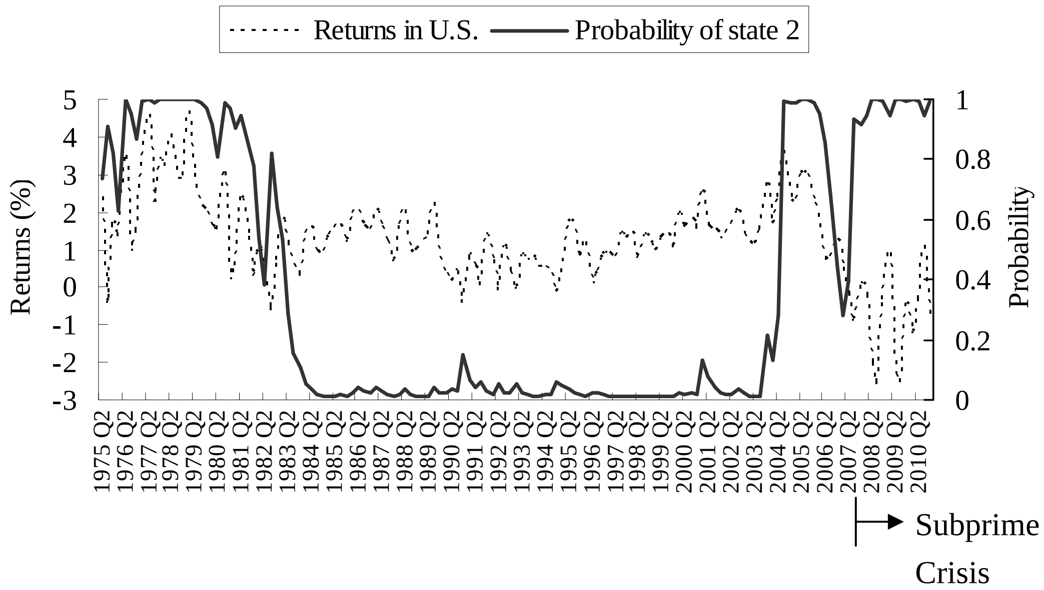

Figure 2. Inferred regime probability of being in state 2 (high-volatility state) estimated from the RS model for the U.S. market

Returns in the U.S. housing market are calculated by the all-transactions house price index, obtained from the Federal Housing Finance Agency (FHFA). The sample period ranges from Quarter 1, 1975 to Quarter 4, 2010. This figure shows that the inferred probability of state 2 (high-volatility state) is close to 1 when the U.S. housing market is volatile. By contrary, this probability approaches to 0 when the market is relatively stable. It is also observed that the RS model clearly identifies the regime transferring to the high-volatility state in the beginning of the subprime crisis, i.e., Quarter 3, 2007. It indicates that the ability of the RS model in capturing the cyclical-volatility property of housing prices is superior.